

B TECH
(SEM II) THEORY EXAMINATION 2018-19
ENGINEERING MATHEMATICS II

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief. 2 x 10 = 20

- a. Solve the differential equation $\frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} + 12y = 0$.
- b. Find the P.I. of $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = e^x \cos x$.
- c. State Rodrigue's formula for Legendre polynomial $P_n(x)$
- d. Prove that $J'_0 = -J_1$.
- e. Find $L\{e^{-t} \sin 2t\}$.
- f. Find $L^{-1}\left\{\frac{1}{(s+2)(s+3)}\right\}$.
- g. Find value of a_0 in Fourier series expansion of $x \sin x$, $0 < x < 2\pi$.
- h. Solve the PDE $(D^2 - 4DD' + 4D'^2)z = 0$.
- i. Classify the PDE $\frac{\partial^2 u}{\partial x^2} + 3 \frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial t^2} = 0$
- j. State one dimensional wave equation.

SECTION B

2. Attempt any three of the following: 10 x 3 = 30

- a. Solve the following simultaneous differential equations:
 $\frac{dx}{dt} - 7x + y = 0, \frac{dy}{dt} - 2x - 5y = 0$.
- b. Solve the following equation in powers series about $x = 0$:
 $2x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - (x+1)y = 0$.
- c. Using Laplace transform, find the solution of the differential equation
 $y'' + 9y = 6 \cos 3t, y(0) = 2, y'(0) = 0$
- d. A periodic function of period 4 is defined as $f(x) = |x|$, $-2 < x < 2$, find its Fourier series expansion.
- e. A string is stretched to two fixed points l apart. Motion is started by displacing the string in the form $u = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement at any time at a distance x from one end at time t is given by
 $u(x,t) = a \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l}$.

SECTION C

3. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Solve $(D^2 + 2D + 1)y = x \cos x$.
- (b) Solve $x \frac{d^2 y}{dx^2} - (2x - 1) \frac{dy}{dx} + (x - 1)y = 0$

4. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Prove that $\int_{-1}^1 P_m(x)P_n(x)dx = 0, m \neq n$.
- (b) Prove that $\frac{d}{dx} [x^n J_n(x)] = x^n J_{n-1}(x)$.

5. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Using Laplace transform, evaluate $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$.
- (b) Using convolution theorem, find $L^{-1} \left\{ \frac{s}{(s^2 + 1)(s^2 + 4)} \right\}$.

6. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Find the Fourier series to represent the function $f(x)$ given by
- $$f(x) = \begin{cases} x, & \text{for } 0 \leq x \leq \pi \\ 2\pi - x, & \text{for } \pi \leq x \leq 2\pi \end{cases}$$
- (b) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \cos x$.

7. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Solve the following equation by the method of separation of variables:
- $$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}, \text{ where } u(0, y) = 8e^{-3y}.$$
- (b) A rectangular plate with insulated surfaces is 8cm wide and so long to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along one short edge $y = 0$ is given by $u(x, 0) = 100 \sin \frac{\pi x}{8}, 0 < x < 8$, while the two long edges $x = 0$ and $x = 8$ as well as the other short edge are kept at $0^\circ C$, show that the steady state temperature at any point of the plate is given by
- $$u(x, y) = 100 \sin \frac{\pi x}{8} e^{-\frac{\pi y}{8}}$$