

B.TECH
(SEM II) THEORY EXAMINATION 2018-19
ENGINEERING MATHEMATICS II

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief. 2 x 10 = 20

- a. Find the particular integral of $(D^3 - 3D^2 + 5)y = e^{2x}$, where $D \equiv \frac{d}{dx}$
- b. Solve the differential equation: $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$
- c. For Bessel's function $J_n(x)$, find the value of $J_{\frac{1}{2}}(x)$.
- d. Express x^2 in terms of Legendre's polynomials.
- e. Find the Laplace transform of $\sin^2 3t$
- f. State first shifting theorem for Laplace transform.
- g. Find the Fourier coefficient a_0 for $f(x) = x \cos x$; $-\pi \leq x \leq \pi$.
- h. Find P.I. of the following partial differential equation: $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$
- i. Write the heat equation in one dimension.
- j. Classify the following partial differential equation as hyperbolic, parabolic or elliptic: $5\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial t} - 2\frac{\partial^2 u}{\partial t^2} = 0$

SECTION B

2. Attempt any three of the following: 10 x 3 = 30

- a. Solve the following simultaneous differential equations:
 $\frac{dx}{dt} + \frac{dy}{dt} + 3x = \sin t$ and $\frac{dx}{dt} + y - x = \cos t$.
- b. For Bessel's function $J_n(x)$, prove that $xJ'_n(x) = -nJ_n(x) + xJ_{n-1}(x)$
- c. Evaluate the following integral using Laplace transform: $\int_0^\infty \frac{e^{-t} \sin \sqrt{3}t}{t} dt$
- d. Obtain the Fourier series for the function $f(x) = x^2$, $-\pi < x < \pi$.
- e. Using the method of separation of variables, solve: $\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t}$.

SECTION C

3. Attempt any one part of the following: 10 x 1 = 10

- (a) Apply the method of variation of parameters to solve the following differential equation: $\frac{d^2y}{dx^2} + a^2y = \sec ax$
- (b) Solve: $(D^2 + D - 2)y = e^x + \cos x$

4. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Solve $2x(1-x)y'' + (5-7x)y' - 3y = 0$ in series in powers of x .
 (b) For Legendre's polynomial $P_n(x)$, prove that $\int_{-1}^1 P_m(x)P_n(x)dx = 0, m \neq n$

5. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Find the Laplace transform of square wave function of period a defined as:

$$F(t) = \begin{cases} 1, & 0 < t < \frac{a}{2} \\ -1, & \frac{a}{2} < t < a \end{cases}$$

- (b) Evaluate by using convolution theorem: $L^{-1}\left\{\frac{p}{(p^2+4)^2}\right\}$

6. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Find the Fourier half range cosine series for the function:

$$f(x) = \begin{cases} 0, & 0 < x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2} < x < \pi \end{cases}$$

- (b) Find the general solution of $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

7. Attempt any *one* part of the following: 10 x 1 = 10

- (a) A string is stretched and fastened to two points l apart so that its ends $x = 0$ and $x = l$ are fixed. It is set vibrating by giving each of its points an initial velocity $g(x)$. If the initial displacement of the string is zero, find the deflection $y(x, t)$ of the vibrating string for $t > 0$.
 (b) Solve the equation $u_{xx} + u_{yy} = 0$, Given that:
 $u(0, y) = u(\pi, y) = 0$ for all y .
 $u(x, 0) = k, 0 < x < \pi$,
 $\lim_{y \rightarrow \infty} u(x, y) = 0, 0 < x < \pi$