PAPER ID-421671

Roll No: | | | | | | | | | | | | | |

BTECH
(SEM II) THEORY EXAMINATION 2021-22
MATHEMATICS II

Time: 3 Hours ***Total Marks: 100***

Note: Attempt all Sections. If you require any missing data, then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2*10 = 20**

Q. No.	Questions	CO
(a)	Solve $((D+1)^3 y = 2e^{-x}$	1
(b)	What are the roots of the indicial equation for the power series solution of the differential equation $2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-3)y=0$	1
(c)	Find the volume of hemisphere.	2
(d)	Examine the convergence of improper integral $\int_2^\infty \frac{1}{(x\,logx)}\,dx$	2
(e)	If f(x) =1, 0<x< π is expanded in half range sine series then find the value of b_n.	3
(f)	Discuss the convergence of sequence $(1, 2^1, 2^2, 2^3, 2^4 \ldots\ldots)$.	3
(g)	Define Harmonic function.	4
(h)	Find the points of invariant of the transformation w =$\frac{2z+3}{z+2}$.	4
(i)	State Cauchy integral theorem.	5
(j)	Discuss the singularity of $\sin(\frac{1}{z-a})$.	5

SECTION B

2. Attempt any *three* of the following: **10*3 = 30**

Q. No	Questions	CO
(a)	Solve $x\frac{d^2y}{dx^2} + (4x^2-1)\frac{dy}{dx} + 4x^3y = 2x^3$	1
(b)	Show that Γ(m). Γ (m + $\frac{1}{2}$) = $\frac{\sqrt{\pi}}{(2)^{2m-1}}$ Γ(2m) where m is positive.	2
(c)	Obtain Fourier series for $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$	3
(d)	Examine the nature of the function $f(z) = \begin{cases} \frac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ in the region including the origin.	4
(e)	$Evaluate\ \frac{1}{2\pi i}\oint_C \frac{z^2-z+1}{z-2}\,dz$, where $C \equiv \lvert z-1\rvert = \frac{1}{2}$	5

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SECTION C

3. Attempt any *one* part of the following: 10*1 = 10

Q. No	Questions	CO
(a)	Solve by change of independent variable method $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$	1
(b)	Solve the equations $t\frac{dy}{dt} + x = 0$ and $t\frac{dx}{dt} + y = 0$ given x (0)=1 and y(-1) = 0	1

4. Attempt any *one* part of the following: 10 *1 = 10

(a)	Analyze the volume contained in the solid region in the 1st octant of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ by applying Dirichlet's Integral	2
(b)	Establish the relation between Beta and Gamma function.	2

5. Attempt any *one* part of the following: 10*1 = 10

(a)	Find half range Fourier sine series for $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$	3
(b)	Examine the series for convergence or divergence $1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \cdots\ldots\ldots\ldots$	3

6. Attempt any *one* part of the following: 10*1 = 10

(a)	Define an analytic function. If $f(z) = u + iv$ is an analytic function find f(z) in terms of z if $u - v = e^x(cosy - siny)$.	4
(b)	Find the image of the circle $\lvert z-1\rvert=1$ in the complex plane under the mapping $w = 1/z$.	4

7. Attempt any *one* part of the following: 10*1 = 10

(a)	Find Laurent series expansion of $\frac{1-cosz}{z^3}$ about the point z = 0 is	5
(b)	Find residue at each pole of the function $\frac{4+3Z}{(Z-2)(Z-3)}$ and hence using Cauchy residue theorem evaluate integral $\oint_C \frac{4+3Z}{(Z-2)(Z-3)}dz$, where C: $\lvert z\rvert = 1$	5