Printed Pages:02 **Sub Code: KAS-103**

Paper Id: **233425** **Roll No.** | | | | | | | | | | |

B. TECH.
(SEM I) THEORY EXAMINATION 2022-23
MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 10 = 20**

a. Find the value of $'b'$ for which the rank of the matrix $A=\begin{bmatrix}1 & 5 & 4\\ 0 & 3 & 2\\ b & 13 & 10\end{bmatrix}$ is 2.pe

b. The Eigen values of A are $2,3,1$ then find the Eigen values of $A^2 + A$.

c. State Roll's Theorem.

d. If $y = \log x^3$ then find y_n.

e. What is the functional relation between $u = \frac{x}{y}$ and $v = \frac{x+y}{x-y}$.

f. Compute an approximate value of $(1.04)^{3.01}$

g. Evaluate $\int_0^1 \int_0^x xy\,dy\,dx$

h. Evaluate the area enclosed between the parabola $y = x^2$ and the straight line $y = x$

i. Prove that if $\vec{u}$ and $\vec{v}$ are irrotational then $\vec{u} \times \vec{v}$ is solenoidal.

j. Find $\nabla(\log r)$ if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is position vector.

SECTION B

2. Attempt any *three* of the following: **10x3=30**

a. Reduce the following matrices to its normal (or canonical) form and find the rank:

$$\begin{bmatrix}1 & 2 & -1 & 4\\ 2 & 3 & 3 & 4\\ 1 & 2 & 3 & 4\\ -1 & -2 & 6 & -7\end{bmatrix}$$

b. If $x = \sin(\sqrt{y})$ then find $(y_n)_0$.

c. A rectangular box, open at the top, is to have a volume of $32\,c.c$. Find the dimensions of the box requiring least material for its construction.

d. Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}}\,dz\,dy\,dx$

e. Find the constants a and b such that the curl of a vector $\vec{A} = (2xy + 3yz)\hat{i} + (x^2 + axz - 4z^2)\hat{j} + (3xy + 2byz)\hat{k}$ is zero.

SECTION C

3. Attempt any *one* part of the following: 10x1=10

a. Find the inverse of the following matrices:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

b. Find the Eigen values and Eigen vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

4. Attempt any *one* part of the following: 10x1=10

a. If $u = f(y-z, z-x, x-y)$, Prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

b. Trace the following curves:
$y^2(2a - x) = x^3$

5. Attempt any *one* part of the following: 10x1=10

a. Expand $\sin^{-1} x$ up to four terms in powers of x.

b. If u, v, w are the roots of the equations $(x-a)^3 + (y-b)^3 + (z-c)^3 = 0$, then find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$.

6. Attempt any *one* part of the following: 10x1=10

a. Change the order of integration in $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2 + y^2}$ hence evaluate the same

b. Evaluate by changing the variables, $\iint_R (x+y)^2\, dx\, dy$ where R is the region bounded by parallelogram $x + y = 0, x + y = 2, 3x - 2y = 0, 3x - 2y = 3$

7. Attempt any *one* part of the following: 10x1=10

a. Evaluate $\iint_S (a^2x^2 + b^2y^2 + c^2z^2)^{1/2}\, dS$ where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$.

b. Find the directional derivative of $\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point $P(1,1,1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$.