

B. TECH.
(SEM I) THEORY EXAMINATION 2022-23
ENGINEERING MATHEMATICS-I

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief.

2 x 10 = 20

- a. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ and $\alpha\delta - \beta\gamma = 1$, then A^{-1}
- b. If the Eigen values of the matrix A are 1,2,3 then find the Eigen values of $(A.I^2)$
- c. State Lagrange's mean value theorem.
- d. What is the eighth derivative of $y = 3x^8$?
- e. If $u = e^x \sin y$ and $v = e^x \cos y$, Evaluate $\frac{\partial(u,v)}{\partial(x,y)}$.
- f. What is the maximum value of function $f(x,y) = 1 - x^2 - y^2$?
- g. What does $\int_0^1 \int_0^{1-x} dy dx$ represent?
- h. Evaluate $\int_0^1 \int_0^x xy dy dx$
- i. Find the $grad \phi$ when ϕ is given by $\phi = 3x^2y - y^3z^2$ at the point $(1, -2, -1)$.
- j. Find the directional derivatives of $\frac{1}{r}$ in the direction of \vec{r} where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

SECTION B

2. Attempt any three of the following:

10x3=30

- a. Find the value of P for which the matrix $A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$ is of rank 1.
- b. If $y = (\sinh^{-1} x)^2$ then find $(y_n)_0$.
- c. Prove by Maclaurin's Theorem that $e^{\sin x} = 1 + x + \frac{x^2}{1.2} - \frac{3.x^4}{1.2.3.4} + \dots$
- d. Evaluate by changing the order of integration $\int_0^1 \int_{2y}^2 e^{x^2} dx dy$
- e. If $\vec{F} = 3y\hat{i} - xz\hat{j} + yz^2\hat{k}$ and S is the surface of the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$, show by using Stoke's theorem $\iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{S} = -20\pi$

SECTION C

3. Attempt any one part of the following: 10x1=10

- a. Investigate, for what values of λ and μ do the system of equations $x + y + z = 6; x + 2y + 3z = 10; x + 2y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) infinite solution
- b. State Cayley-Hamilton Theorem and verify it for the matrix;

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 9 & 2 & 0 \\ 5 & 0 & 3 \end{pmatrix} \quad \text{Also find } A^{-1}.$$

4. Attempt any one part of the following: 10x1=10

- a. If $x^x y^y z^z = c$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$
- b. Trace the curve: $x^3 + y^3 = 3axy$

5. Attempt any one part of the following: 10x1=10

- a. Find the possible % error in computing the parallel resistance r of three resistances r_1, r_2, r_3 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ if r_1, r_2, r_3 are each in error by $+1.2\%$.
- b. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

6. Attempt any one part of the following: 10x1=10

- a. Evaluate $\iint_R (x-y)^4 e^{x+y} dx dy$ where R is the square with vertices at $(1,0), (2,1), (1,2)$ and $(0,1)$.
- b. Evaluate: $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dy dx dz$

7. Attempt any one part of the following: 10x1=10

- a. If $\vec{F} = (\vec{a} \cdot \vec{r}) \vec{r}$, where \vec{a} is a constant vector, find $\text{curl} \vec{F}$ and prove that it is perpendicular to \vec{a} .
- b. Evaluate $\iint_S (a^2 x^2 + b^2 y^2 + c^2 z^2)^{-1/2} dS$ where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$.