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Paper Id: 233439 Roll No.

B.TECH.
(SEM I) THEORY EXAMINATION 2022-23
ENGINEERING MATHEMATICS-I

Time: 3 Hours ***Total Marks: 70***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 7 = 14

(a) If $y = sinhx$ then find y_{2n} and y_{2n+1}.

(b) What is the asymptote of the curve $y^2(2a-x)=x^3$?

(c) Find approximately $\frac{1}{2.1}$

(d) Find the product of the Eigen values of the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 3 & -1\\ 0 & -1 & 3\end{bmatrix}$

(e) Evaluate: $\beta\left(\frac{1}{2},\frac{1}{2}\right)$

(f) Find the directional derivative of $\phi = 2x^2y+3xy^3$ in the direction of $3\hat{i}+4\hat{j}$ at the point $(1,1)$.

(g) Find $\vec{\nabla}(\log r)$ if $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ is position vector.

SECTION B

2. Attempt any *three* of the following: 7 x 3 = 21

(a) If $y=e^{m\cos^{-1}x}$ then find $(y_n)_0$.

(b) Find the minimum value of $x^2+y^2+z^2$, given that $ax+by+cz=p$

(c) Reduce the following matrices to its normal (or canonical) form and find the rank:

$$\begin{bmatrix}1 & 2 & 1 & 0\\ -2 & 4 & 3 & 0\\ 1 & 0 & 2 & -8\end{bmatrix}$$

(d) Evaluate $\iint y\,dx\,dy$ over the part of the plane bounded by the line $y=x$ and the parabola $y=4x-x^2$

(e) Verify Stoke's theorem for $\vec{F}=(x^2+y^2)\hat{i}-2xy\,\hat{j}$ taken round the rectangle bounded by the lines $x=\pm a, y=0, y=b$.

SECTION C

3. **Attempt any *one* part of the following:** **7 x 1 = 7**

(a) If $u=\tan^{-1}\frac{x^3+y^3}{x-y}$ prove that (i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\sin 4u-\sin 2u=2\cos 3u\sin u$

(b) Trace the curve: $xy^2=a^2(a-x)$

4. **Attempt any *one* part of the following:** **7 x 1 = 7**

(a) Prove that $JJ'=1$, where J is Jacobian.

(b) Expand $e^x\sin y$ in powers of x and y as far as terms of third degree.

5. **Attempt any *one* part of the following:** **7 x 1 = 7**

(a) For what values of k, the equations $x+y+z=1; 2x+y+4z=k; 4x+y+10z=k^2$ have a solution and solve them completely in each case.

(b) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$

6. **Attempt any *one* part of the following:** **7 x 1 = 7**

(a) Evaluate the integral $\iiint_V x^{l-1}y^{m-1}z^{n-1}\,dxdydz$ where x, y, z are all positive but limited by the condition $\left(\frac{x}{a}\right)^p+\left(\frac{y}{b}\right)^q+\left(\frac{z}{c}\right)^r\le 1$

(b) Change the order of integration in $I=\int_0^1\int_{x^2}^{2-x} xy\,dydx$ and hence evaluate the same.

7. **Attempt any *one* part of the following:** **7 x 1 = 7**

(a) Prove that $(y^2-z^2+3yz-2x)\hat{i}+(3xz+2xy)\hat{j}+(3xy-2xz+2z)\hat{k}$ is both solenoidal and irrotational.

(b) Using the Green's theorem, evaluate the integral: $\int_C (xy\,dy-y^2\,dx)$, where C is the square cut form the first quadrant by the line $x=1, y=1$.