PAPER ID-311450

Subject Code: KAS103

Roll No: | | | | | | | | | | | | | |

BTECH
(SEM I) THEORY EXAMINATION 2023-24
MATHEMATICS-I

TIME: 3HRS **M.MARKS: 100**

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 10 = 20**

Qno.	Question	Marks	CO
a.	Obtain the Eigen values of $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 3 & 4 & 5 \end{bmatrix}$	2	1
b.	Define orthogonal matrix.	2	1
c.	State Lagrange's Mean value theorem	2	2
d.	Write statement of Leibnitz theorem.	2	2
e.	If $u = (x^2 + y^2 + z^2)$ then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$	2	3
f.	If $pv^2 = k$ and the relative error in p and v are respectively 0.05 and 0.025, show that the error in k is 10%.	2	3
g.	Evaluate $\int_0^1 \int_0^y dx\, dy$.	2	4
h.	Write the formula of Area and volume by integration.	2	4
i.	Find unit normal to the surface $z = 2xy$ at the point $(2,1,4)$ *is* ?	2	5
j.	Find the Value of K for which a vector $\vec{F} = (Kx + 4y^2z)i + (x^3 sinz - 3y)j - (e^x + 4cosx^2y)k$ is a Solenoidal Vector.	2	5

SECTION B

2. Attempt any *three* of the following: **10x 3 = 30**

a.	Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify Cayley Hamilton theorem.	10	1
b.	If $y = (sin^{-1}x)^2$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2)y_n = 0$	10	2
c.	If u, v *and* w *the roots of equation* $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ *in* λ, *find* $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	10	3
d.	Apply Dirichlet's integral to find the Mass of an octane of the ellipsoid $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) + \left(\frac{z^2}{c^2}\right) = 1$, the density at any point being $\rho = kxyz$.	10	4
e.	Verify Gauss Divergence theorem for $\vec{F} = 4xzi - y^2j + yzk$ taken over the cube bounded by the planes: $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1.$	10	5

SECTION C

3. Attempt any *one* part of the following: **10x 1 = 10**

a.	Find the rank of matrix reducing into Canonical form for $A = \begin{bmatrix} 3 & 2 & -1 \\ 4 & 2 & 6 \\ 7 & 4 & 5 \end{bmatrix}$	10	1

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b.	Test the consistency and solve the following system of equations $2x - y + 3z = 8, \quad -x + 2y + z = 4, \quad 3z + y - 4z = 0.$	10	1

4. Attempt any *one* part of the following: **10x 1 = 10**

a.	If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$ prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$	10	2
b.	Verify Rolle's theorem for the function $f(x)= x^2-x+1$ on [0,1].	10	2

5. Attempt any *one* part of the following: **10x 1 = 10**

a.	If $u_1 = \frac{x_2x_3}{x_1}, u_2 = \frac{x_3x_1}{x_2}, u_3 = \frac{x_1x_2}{x_3}$, find $\frac{\partial(u_1, u_2, u_3)}{\partial(x_1, x_2, x_3)}$.	10	3
b.	Verify Euler theorem for $u = log\left(\frac{x^4+y^4}{x+y}\right)$.	10	3

6. Attempt any *one* part of the following: **10x 1 = 10**

a.	Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2 + y^2)\, dxdy$ by changing into polar Co-ordinates.	10	4
b.	Evaluate the area enclosed between the parabola $y = x^2$ and the straight line y=x.	10	4

7. Attempt any *one* part of the following: **10x 1 = 10**

a.	Prove that $(y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational.	10	5
b.	Using Green's Theorem evaluate: $\int_C (x^2 + xy)dx + (x^2 + y^2)dy$, where C is the square formed by the lines $x = \pm 1, y = \pm 1$.	10	5