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BTECH
(SEM I) THEORY EXAMINATION 2024-25
MATHEMATICS-I

TIME: 3 HRS**M.MARKS: 100****Note: 1.** Attempt all Sections. If require any missing data, then choose suitably.**SECTION A****1. Attempt all questions in brief.****2 x 10 = 20**

Qno.	Question	Marks	CO
a.	Find the characteristic values of the matrix $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$.	2	1
b.	Show that $\frac{1}{2} \begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is a unitary matrix.	2	1
c.	State Rolle's theorem.	2	2
d.	Find y_n , if $y = x^2 e^x$.	2	2
e.	State Taylor's theorem for a function of two variables.	2	3
f.	Find $\frac{du}{dt}$ as a total derivative if $u = x^3 + y^3$ and $x = a \cos t, y = b \sin t$.	2	3
g.	Determine the area enclosed between the parabola $y = x^2$ and the straight line $y = x$.	2	4
h.	Solve $\int_0^1 \int_0^{x^2} e^{y/x} dy dx$.	2	4
i.	Discuss the value of m if $\vec{F} = m\hat{x}i - 5y\hat{j} + 2z\hat{k}$ is a solenoidal vector.	2	5
j.	Discuss the value of $\text{curl } \vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.	2	5

SECTION B**2. Attempt any three of the following:**

a.	Determine the values of λ and μ such that the system $2x - 5y + 2z = 8, 2x + 4y + 6z = 5, x + 2y + \lambda z = \mu$ has i. No solution ii. A unique solution iii. Infinite number of solutions.	10	1
b.	Discuss Lagrange's mean value theorem to find the value of c for the function: $f(x) = (x-1)(x-2)(x-3)$ in $[0,4]$.	10	2
c.	If $u = \cos e^{-1} \left(\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}} \right)^{1/2}$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144} (13 + \tan^2 u)$.	10	3
d.	Evaluate the following integral by changing the order of integration: $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$.	10	4
e.	Using Green's theorem, determine the value of $\int_C (2y^2 dx + 3x dy)$ where C is the boundary of the closed region bounded by $y = x, y = x^2$.	10	5



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SECTION C

3. Attempt any one part of the following:

a.	Calculate the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$	10	1
b.	Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute A^{-1} . Also find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.	10	1

4. Attempt any one part of the following:

a.	If $y = \sin(a \sin^{-1} x)$, find $y_n(0)$.	10	2
b.	Find the envelope of the family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a+b=c$.	10	2

5. Attempt any one part of the following:

a.	If $x + y + z = u$, $y + z = uv$, $z = uvw$, then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.	10	3
b.	The sum of three positive numbers is constant. Prove that their product is maximum when they are equal.	10	3

6. Attempt any one part of the following:

a.	Evaluate by changing the variables, $\iint_R (x+y)^2 dx dy$ where R is the region bounded by the parallelogram $x+y=0, x+y=2, 3x-2y=0$ and $3x-2y=3$.	10	4
b.	A triangular prism is formed by planes whose equations are $ay = bx, y = 0$ and $x = a$. Find the volume of the prism between the planes $z = 0$ and surface $z = c + xy$.	10	4

7. Attempt any one part of the following:

a.	Verify Stoke's theorem, for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.	10	5
b.	Apply Divergence theorem to evaluate $\iiint_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = 4x^3\hat{i} - x^2y\hat{j} + x^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by the planes $z = 0$ and $z = b$.	10	5