PAPER ID-310128

Subject Code: MTEC101

Roll No: | | | | | | | | | | | | | |

MTECH
(SEM I) THEORY EXAMINATION 2025-26
ADVANCED ENGINEERING MATHEMATICS

TIME: 3 HRS **M.MARKS: 70**

Note: Attempt all Sections. In case of any missing data; choose suitably.

SECTION A

1. Attempt *all* questions in brief. **02 x 7 = 14**

Q no.	Question
a.	Define transient and steady states.
b.	If X and Y are two random variables having joint density function $f(x,y)=\begin{cases}\frac{1}{8}(6-x-y); 0\le x<2,\ 2\le y<4\\ 0, \quad \text{Otherwise}\end{cases}$ then find $P(X+Y<3)$.
c.	State bivariate normal distribution.
d.	Describe the M/M/1 queuing model and provide an example of its application in a real-world scenario.
e.	Find the moment generating function of the random variable whose moments are: $\mu_r' = (r+1)!2^r$.
f.	If x, y, z are linearly independent vectors, prove that $x+y$, $y+z$, $z+x$ are also linearly independent.
g.	Show power spectal density is even function.

SECTION B

2. Attempt any *three* of the following: **07 x 3 = 21**

Q no.	Question
a.	Consider the Probability Density Function (PDF) for a continuous random variable X $$f_X(x)=\begin{cases}\frac{3}{7}x^2, & 1\le x\le 2,\\ 0, & \text{otherwise.}\end{cases}$$ Calculate the cumulative distribution function (CDF) of X and use it to find the probability that X lies in the interval $[1.5, 3]$.
b.	Show that the power spectrum of an SSS process $x(t)$ equals $S(w)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 G(x_1,x_2;\omega)dx_1dx_2$, where $G(x_1,x_2;\omega)$ is the Fourier transform in the variable τ of the second-order density $f(x_1,x_2;\tau)$ of $x(t)$.
c.	Patients visit a dentist at the rate of 8/hour (Poisson) and the service times are 11/hour (exponentially distributed). Find the expected waiting time in the system. If the dentist wishes to have a finite queue length model, find N for which time in the system is less than 15 minutes.
d.	A system has m components that become ON and OFF independently. Suppose the ON and OFF processes are independent Poisson processes with parameter λ and μ, respectively. Determine the steady state probability of exactly k components being ON, $k = 0,1,2.....\ m$.
e.	Find the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ where $T(1,0) = (1,-1,0)$ $T(0,1) = (0,2,1)$.

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SECTION C

3. Attempt any *one* part of the following: 07 x 1 = 07

Q no.	Question
a.	If V is finite dimensional and f is a homomorphism of V onto V prove that f must be one-one and so, an isomorphism.
b.	The joint probability density function of the two random variables X and Y is given by $f(x,y)=\frac{9(1+x+y)}{2(1+x)^4(1+y)^4};0\le x<\infty,\ 0\le y<\infty$. Find the marginal distributions of X and Y, and the conditional distribution of Y for $X=x$.

4. Attempt any *one* part of the following: 07 x 1 = 07

Q no.	Question
a.	State and prove the central limit theorem.
b.	Find the dual basis of the basis B={(1,0,0),(1,1,0),(1,1,1)} of the vector space $R^3(R)$.

5. Attempt any *one* part of the following: 07 x 1 = 07

Q no.	Question
a.	State and establish the properties of Poisson process.
b.	Let V be the vector space of all 2×2 matrices over the field F. Prove that V has dimension 4 by exhibiting a basis for V which has 4 elements.

6. Attempt any *one* part of the following: 07 x 1 = 07

Q no.	Question
a.	State and prove Chapman-Kolmogorov Equation.
b.	The probability density function of the random variable X follows the probability law: $p(x)=\frac{1}{2\theta}\exp\left(-\frac{\lvert x-\theta\rvert}{\theta}\right),\ -\infty<x<\infty$. Find moment generating function of X. Hence or otherwise find $E(X)$ and $V(X)$.

7. Attempt any *one* part of the following: 07 x 1 = 07

Q no.	Question
a.	If V and U finite dimensional vector space over F, then show $\dim Hom(V,U)=\dim V.\dim U$
b.	Show that an $M/G/1$ queueing system is transient if the mean value of the number of customers arrive during a service period is greater than one.